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Abstracts

Özlem Beyarslan
Model Theory of Fields with Virtually Free Group Action

This is joint work with Piotr Kowalski. A G-field is a field, together with an acion of a group G by field automorphisms. If an axiomatization for the class of existentially closed G-fields exists we call the resulting theory G-TCF. If G is the group of integers then G-TCF exists and coincides with ACFA. More generally when G is free on n-generators then the theory of existentially closed models is ACFA_n. G-TCF also exists when G is finite. Our main theorem says that G-TCF exists when G is virtually free. We also give criteria for the simplicity of G-TCF.

Martin Hils
Definable equivariant retractions onto skeleta in non-archimedean geometry

For a quasi-projective variety V over a non-archimedean valued field, Hrushovski and Loeser recently introduced a pro-definable space $\widehat{V}$, the *stable completion* of V, which is a model-theoretic analogue of the Berkovich analytification of V. They showed that $\widehat{V}$ admits a pro-definable strong deformation retraction onto a skeleton, i.e., onto a space which is internal to the value group and thus piecewise linear. If the underlying variety is an algebraic group, the group naturally acts on its stable completion by translation. In the talk, we will sketch various ways to construct an S-equivariant pro-definable strong deformation retraction of $\widehat{S}$ onto a skeleton, in case S is a semiabelian variety. This is joint work with Ehud Hrushovski and Pierre Simon.

Ehud Hrushovski
A tour of globally valued fields

This will be a gentle introduction to the emerging model theory of GVFs, using a number of specific formulas as examples.

Rémi Jaoui

Disintegrated differential equations and mixing Anosov flows

The classification of minimal types in differentially closed fields is beautiful evidence for the effectiveness of geometric stability theory in the study of some noetherian geometric contexts that were previously inaccessible. However for a given nonintegrable differential equation, it is often hard to determine the properies of the correspoinding definable set in DCF_0.

On the other hand, for many interesting examples of noninntegrable equations, Anosov has isolated some very concrete properties reflecting the "chaotic" nature of the corresponding flow acting on the set of initial conditions.

Starting with a differential equation over the field of real numbers, I will explain how to witness disintegration of its generic type (in DCF_0) from the dynamical properties of the associated flow. Then I will apply this technique to study certain differential equations describing the geodesic motion on an algebraically presented Riemannian manifold with negative curvature.

These results were obtained during my Ph.D. and shortly after.

Itay Kaplan

On PC-exact saturation

(joint work with Nick Ramsey and Saharon Shelah) A theory T is said to have exact saturation at a (usually singular) cardinal κ if there is a model which is κ- saturated but not κ^+- saturated. T has PC-exact saturation at κ if for every T_1 containing T (of the same cardinality), there is a model of T_1 whose reduct to the language of T is κ-saturated but not κ^+-saturated. In a previous work (with Shelah and Simon) we identified exact saturation in simple in NIP theories. By work of Shelah and Malliaris, if a theory has SOP2 then it does not have PC exact saturation. In this talk I will discuss PC exact saturation in the context of simple theories, and also the local analog.

Krzysztof Krupinski

Boundedness and absoluteness of some dynamical invariants in model theory

Let $\mathbb{C}$ be a monster model of a theory T. With a given $\emptyset$-type-definable set X one can associate the flow $(Aut(\mathbb{C}), S_X(\mathbb{C}))$, where $S_X(\mathbb{C})$ is the compact space of all global types concentrated on X. Then topological dynamics yields various useful notions such as the Ellis semigroup EL of this flow together with minimal left ideals of EL which in turn are decomposed as disjoint unions of certain isomorphic groups whose isomorphism type is sometimes called the Ellis group of the flow in question. These notions turned out to be essential to understand the complexity of Galois groups of first order theories and "spaces" of strong types. Now, we investigate whether these notions have a model-theoretic nature, i.e. whether they are independent of the choice of the monster model $\mathbb{C}$; if a notion does not depend on $\mathbb{C}$, we call it absolute. We introduce a notion of content of a tuple of types (which in a sense generalizes fundamental orders in stable theories), and, using it, we show that the Ellis group of the flow $(Aut(\mathbb{C}), S_X(\mathbb{C}))$ is always absolute, and we provide an absolute bound on its size. In contrast, minimal left ideals of EL do not have to be of bounded size, but if they are, we also show the appropriate version of absoluteness. Moreover, we characterize when minimal left ideals of EL are of bounded size (which is particularly interesting in NIP theories). All of this comes from my joint paper with Ludomir Newelski and Pierre Simon.

François Loeser

Non-archimedean integrals as limits of complex integrals

Chambert-Loir and Ducros have recently developed a theory of real differential forms, integration and currents on Berkovich spaces which is parallel to the the classical theory of differential forms on complex spaces. We will report on joint work with Ducros and Hrushovski, showing that such non-archimedean forms and integrals arise naturally as limits of their archimedean counterparts when one studies the asymptotic behaviour of families of complex varieties on the punctured disc.

Dugald Macpherson

Definably simple groups in valued fields

I will discuss joint work with Gismatullin, Halupczok, and Simonetta on the following problem: given a henselian valued field of characteristic 0, possibly equipped with analytic structure, describe the possibilities for a definable group G in the valued field sort which is definably almost simple, that is, has no proper infinite definable normal subgroups. We also have results for an algebraically closed valued field K in characteristic p, but assuming also that the group is a definable subgroup of $\mathrm{GL}(n, K)$.

Rahim Moosa

D-varieties and the Dixmier-Moeglin Equivalence

About four years ago, a new application of the model theory of differentially closed fields arose. The target was the Dixmier-Moeglin equivalence problem (DME) in noncommutative affine algebras, as well as a variant for commutative Poisson algebras. It has become clear that the structure of D-varieties (i.e., finite-dimensional types in DCF), and D-groups, can be used to both prove the DME and produce counterexamples in various settings. There have been a handful of papers exploring and exploiting this connection. I will give an overview of this body of work.

Joel Nagloo

Towards Strong Minimality and the Fuchsian Triangle Groups

From the work of Freitag and Scanlon, we have that the ODEs satisfied by the Hauptmoduls of arithmetic subgroups of $SL_2(\mathbb{Z})$ are strongly minimal and geometrically trivial. A challenge is to now show that same is true of ODEs satisfied by the Hauptmoduls of all (remaining) Fuchsian triangle groups. The aim of this talk is to both explain why this an interesting/important problem and also to discuss some of the progress made so far.

Alf Onshuus

On groups definable in geometric fields

Geometric fields are fields where model theoretic algebraic closure is the same as field theoretic algebraic closure, and which eliminate exist infinity. Hrushovski and Pillay proved that any group G definable in such a field is related via a group configuration theorem with an algebraic group H. We will talk about how close this relationship is in various cases. In particular we will use a local version of Hrushovkski's Stabilizer Theorem to find an isogeny between G and a subgroup of H when G is definably amenable.

Jonathan Pila

Ax-Schanuel for Shimura varieties

In 1971, Ax proved functional versions of Scahanuel's conjecture for the expoential function, including in the setting of differential fields. This result is known as "Ax-Schanuel". I will describe joint work with N. Mok and J. Tsimerman proving analogues of Ax-Schanuel for Shimura varieties.

Ya'acov Peterzil

Strongly minimal groups in o-minimal structures

Let G be a definable two-dimensional group in an o-minimal structure M and let D be a strongly minimal expansion of G, whose atomic relations are definable in M. We prove that if D is not locally modular then G is definably isomorphic to a one dimensional algebraic group A over a D-definable algebraically closed field K. Moreover, D is precisely the structure which K induces on A.

The result generalizes a theorem of Hasson and Kowalski on expansions of $(\mathbf{C}, +)$ and gives another indication to the so-called Zilber's Trichotomy conjecture, for strongly minimal structures that are definable in o-minimal ones. The proof of the result combines the geometric constraints of o-minimality, together with the combinatorial restrictions of strong minimality, in order to recover "complex intersection theory" which allows us to define a field.
(joint work with Pantelis Eleftheriou and Assaf Hasson)

Thomas Scanlon
What might model theory say about Hodge theory?

The Hodge decomposition of the cohomology of a smooth projective complex algebraic variety gives refined invariants based on the real structure of the variety. Though these data are defined analytically, they carry algebraic and even arithmetic information. Indeed, many of the special point problems of André-Oort type may be understood Hodge theoretically and the celebrated Hodge Conjecture asserts that so-called Hodge classes must be of algebraic origin. Mediated by definability in o-minimal structures and the theory of differentially closed fields, ideas from model theory can inform Hodge theory. In this lecture, I will report on various points of contact I see between model theory and Hodge theory, including on an on-going project with Chuck Doran on finiteness results for intersections with jump loci in moduli spaces of K3 surfaces, on recent work of Bakker and Tsimerman on an Ax-Schanuel theorem for variations of Hodge structures, and on non-linear differential algebraic maps associated to period mappings.

Tamara Servi
On local interdefinability of (real and complex) analytic functions

Given two (real or complex) analytic functions f and g, it is not sensible in general to ask whether they are first-order interdefinable as total functions (think of the sine function). It does make sense to ask whether f and g are locally interdefinable in the context of o-minimal structures. For example, the real exponential function and the sine function are not locally interdefinable [Bianconi]. The same holds for complex exponentiation and any Weierstrass $\wp$-function [Jones, Kirby, Servi]. Two Weierstrass $\wp$-functions are locally interdefinable if and only if one can be obtained from the other by isogeny and Schwarz reflection [Jones, Kirby, Servi]. There are complex analytic functions which are locally interdefinable and which cannot be obtained from one another by elementary operations such as Schwarz reflection, composition, derivation and extracting implicit functions [Jones, Kirby, Le Gal, Servi]. In the case of real analytic functions, it is possible to give an analytic characterisation of all the functions g which are locally definable from f [Le Gal, Servi, Vieillard-Baron]. The proofs of the aforementioned results rely on the interaction between methods from functional transcendence, resolution of singularities and model theory.

Joszef Solymosi
Structure from density

I will mention some combinatorial problems related to dense structures. For example, if n points in the plane determine cn^2 lines with at least three points on them, then one expects that many of the points are in a special position. Potential tools to analyse such arrangements are Regularity Lemmas and combinatorial variants of the Group Configuration Theorem. Unfortunately even these powerful techniques are not enough to answer some simple questions. For example in the point-line arrangement described above, one would expect that if n is large enough then some (say 10) points are on an elliptic curve, but this problem seems to be out of reach.

Katrin Tent

Ampleness in strongly minimal structures

The notion of ampleness captures essential properties of projective spaces over fields. It is natural to ask whether any sufficiently ample strongly minimal set arises from an algebraically closed field. In this talk I will explain the question and survey recent results on ample strongly minimal structures.

Lou van den Dries

Abraham Robinson's legacy in model theory and its applications

Julia Wolf

The structure of stable sets

Dimitri Wyss

p-adic Integration along the Hitchin Fibration and Applications

By the work of Weil, Batyrev and Denef-Loeser one can use p-adic integration to compute the (stringy) number of rational points of a smooth or mildly singular algebraic variety X. One advantage of this approach is, that one can sometimes avoid the "bad locus" of X. We apply this idea to the moduli space M of G-Higgs bundles and obtain global invariants of M by considering only the generic fibers of the Hitchin fibration. For $G = \mathrm{SL}_n$, PGL_n this gives a proof of the topological mirror symmetry conjecture of Hausel-Thaddeus. If time permits I will sketch for general G a connection with the geometric stabilization theorem of Ngô. This is joint work with Michael Groechenig and Paul Ziegler.